Filter IHCP Three-Dimensional Solution for Large Flat Plates and Small to Moderate Dimensionless Times

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The heat flux is determined for a large flat plate that is heated at $x = 0$ with a heat flux that varies both in the y- and z-directions as well as with time. The plate is homogeneous and insulated at $x = L$. Many temperatures are measured in a checkerboard array at this $x = L$ surface. The objective is to determine the heat flux and temperature distributions at the x = 0 utilizing the transient measurements at $x = L$. In this analysis the heated surface is modeled with a large number of square surfaces resembling a checkerboard; each square element is considered to be $2a$ by $2a$ in dimensions, where $a = L$. The heating is considered to be uniform over each of surface squares but each of these elements has independent time-varying heat fluxes. Because the dimensionless time steps are small is "large," the greatest contribution to heating at a given location at the $x = L$ surface is local in time and space. This observation leads to the local filter solution of this problem.

This analysis uses analytical solutions for a basic building block which is shown in Fig. 1. This figure shows ¼ of a basic element. The boundary conditions at $y = b$ and $z = b$ are not important for the dimensionless times of interest but are shown as isothermal in Fig. 1. Dimensionless temperatures are shown in Table 1 for the center points of some square elements surrounding the point at $y = z = 0$; each is at the insulated surface, $x = L$. The dimensionless time step, $\Delta\tilde{t}_L = \alpha\Delta t / L^2$ = 0.06. At time $\tilde{t}_L = \alpha t / L^2$ = 0.06 the dimensionless temperature at $x = L$, $y = z = 0$ in Table 1 is 0.0007823 which is very small compared to the opposite surface temperature of 0.2762112. Because the temperature at that time is so small compared to the surface (that is, lags well behind the surface temperature), some method of regularization is needed. There are many such methods including function specification which is used in this work. Also the lack of much response in the surrounding elements (see the second and third columns in Table 1) suggests that the heat effects are local in space; it may not be necessary to consider the whole surface and not the complete time domain in a single computation.

If this problem is solved using a finite control volume or finite element, a great many nodes are needed to insure adequate accuracy. In general such a method would be the preferred method of solution. However, using filter concepts [1]extended to this 3D problem, the computations can be greatly reduced and the resulting computer code can be more accessible to non-specialists.

Reference

1. J.V. Beck, Filter solutions for the nonlinear inverse heat conduction problem, *Inverse Problems in Science and Engineering*, 16, 3-20, (2008).

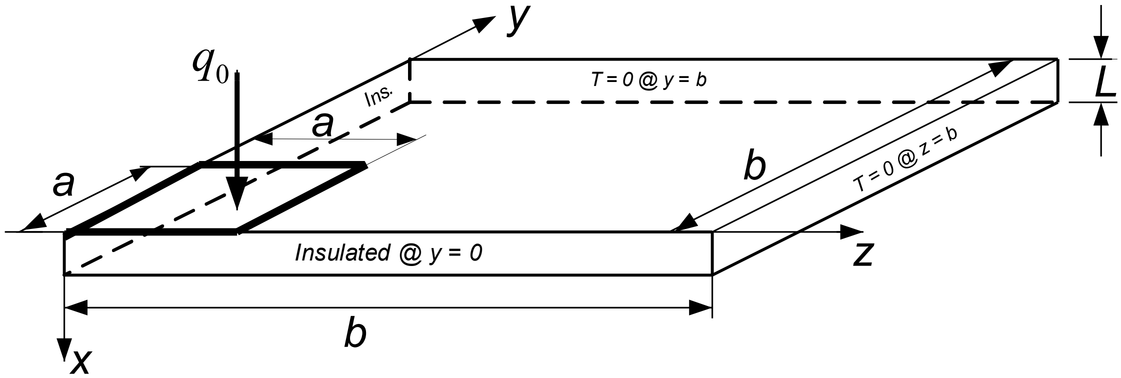

Fig. 1. Three dimensional geometry for uniform in time and space heating at the x = 0 surface over ($0 < y < a,\ 0 < z < a$). Otherwise the surface at $x = 0$ is adiabatic as is the $x = L$ surface. The boundaries at $y = 0$ and $z = 0$ are adiabatic and the boundaries at $y = b$ and $z = b$ are held at zero degrees. The distance b is large compared to a.

Table 1. Numerical values for the transient temperatures for building block, $a/L = 1$, $b/L = 20$

$\tilde{t}_L = \frac{\alpha t}{L^2}$	$\frac{T(L,0,0,t)}{q_0 L / k}$	$\frac{T(L,2L,0,t)}{q_0 L / k}$	$\frac{T(L,2L,2L,t)}{q_0 L / k}$	$\frac{T(L,4L,0,t)}{q_0 L / k}$	$\frac{T(L,4L,2L,t)}{q_0 L / k}$
0.060	0.0007823	0.0000008	0.0000000	0.0000000	-0.0000000
0.120	0.0142051	0.0001681	0.0000025	0.0000000	-0.0000000
0.180	0.0424801	0.0012312	0.0000448	0.0000000	0.0000000
0.240	0.0768402	0.0036462	0.0002179	0.0000001	0.0000000
0.300	0.1120313	0.0073381	0.0006088	0.0000012	0.0000001
0.360	0.1458385	0.0120482	0.0012683	0.0000062	0.0000008
0.420	0.1775024	0.0175166	0.0022155	0.0000211	0.0000035
0.480	0.2068883	0.0235315	0.0034491	0.0000540	0.0000102
0.540	0.2341102	0.0299321	0.0049560	0.0001140	0.0000245
0.600	0.2593661	0.0365989	0.0067175	0.0002105	0.0000500
0.660	0.2828683	0.0434433	0.0087121	0.0003516	0.0000913
0.720	0.3048158	0.0503992	0.0109179	0.0005446	0.0001526
0.780	0.3253856	0.0574169	0.0133130	0.0007951	0.0002382
0.840	0.3447315	0.0644586	0.0158769	0.0011074	0.0003520
0.900	0.3629857	0.0714954	0.0185902	0.0014846	0.0004976
0.960	0.3802616	0.0785052	0.0214349	0.0019285	0.0006778
1.020	0.3966568	0.0854708	0.0243948	0.0024401	0.0008953
1.080	0.4122555	0.0923791	0.0274548	0.0030193	0.0011520
1.140	0.4271306	0.0992203	0.0306014	0.0036655	0.0014493
1.200	0.4413457	0.1059866	0.0338224	0.0043776	0.0017884